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MAKRO

The new Danish macroeconomic model MAKRO

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1 Introduction

The Danish ministries of finance and economic affairs have set up an independent working group with the purpose of building a new macroeconomic model based on up-to-date developments both in the academic literature and in applied work internationally. The name of the new model is MAKRO. The development project was launched in the spring of 2017 and will last for 3 years, under the auspices of the Danish research institute DREAM. MAKRO must provide a sufficiently detailed framework for the ministries' combined short- and long term projections. These projections are used to assess the medium term budgetary outlook and the sustainability of fiscal policy, taking into account changes in fiscal policy within the planning horizon (currently 2025, after which fiscal policy is kept “neutral”) and long term demographic developments (until year 2100). *Thus*, the new model has to be a relatively large-scale macroeconomic model that is both theoretically well-founded and empirically based.

To credibly assess both fiscal sustainability and the structural budget balance, MAKRO must be capable of analyzing changes to the labor supply. To do that, it is crucial to model the age dependent labor supply and forward-looking behavior (note that forward looking is not synonymous with hyper-rational expectations). This is one of the main reasons for switching framework from a traditional SEM (Structural Econometric Model) to a new model based on a forward looking overlapping generations (OLG) framework. As an example consider the retirement age. An important aspect of the Danish economy is that the retirement age as a default increases with the life expectancy. A change in the retirement age is announced at least 15 years prior to a cohort's retirement. Forward looking behavior and planning will affect the households' aggregate savings, consumption, and labor supply even before the actual retirement age is changed.

There already exists a forward looking OLG model of the Danish economy, DREAM. It is a structural model. It can describe the structural effects of a change in the retirement age, but as a structural model it cannot evaluate the dynamic response. The dynamic response is important in practical policy modeling. The model currently used by the Danish Ministry of Finance, ADAM, lacks forward looking expectations and has no explicit age-dependent household decisions. On the other hand it has many properties that a workhorse model for the Ministry of Finance must have. It is a large model and produces data output that serves as a data-repository for projections of macroeconomic variables including detailed government incomes and expenditures. These model projections are both consistent with accounting identities and with the most recent published data from the Danish National Accounts. An Input/Output system provides a consistent framework for disaggregating different goods, services, and production sectors such that indirect taxation can be evaluated. All these properties must be carried over to MAKRO.

MAKRO is a hybrid model in more than one sense. The aim is to build a model that sensibly describes both the short and the long run. In this sense, the model is a hybrid between short-run models and long-run structural models. Short-run models, such as DSGE (Dynamic Stochastic General Equilibrium) models or SEMs, typically describe the rigidities and imperfections that determine how the economy reacts to short-run fluctuations. Economic models of the long run typically attempt to explain structural changes based on factors such as demographics, productivity, and foreign demand. The long run models are often within the CGE (Computable General Equilibrium) tradition. The current prototype of MAKRO is a hybrid between two models: 1) a relatively simple New Keynesian model with short run rigidities (Calvo pricing, Phillips curve, habit formation, rule of thumb consumers), and 2) a fully specified long-run model with overlapping generations households (OLG) that capture demographics and life-cycle savings decisions etc., multiple production sectors with installation costs, and an Armington model of exports and imports. At the time of writing, we are working on richer short-run descriptions of the labor market and financial markets.

MAKRO is also a hybrid between modern DSGE models and traditional SEMs. DSGE models are micro-founded models based on rational agents that optimize under uncertainty, with model parameters that are typically estimated using a likelihood or structural vector autoregression (SVAR) approach to system estimation. DSGE models are characterized by a high degree of theoretical consistency. Modern SEMs are generally also micro-founded to some extend (this is the case for the Danish ADAM model), but microfoundations are rarely as stringent as in DSGE models. Rather, SEMs are typically estimated equation by equation using error correction specifications, rather than as as system. The estimation strategy for MAKRO is a compromise between the two methodologies, and while the model draws inspiration from the DSGE literature there is no uncertainty in the model.

Outside of academia the development of DSGE models is led by central banks, and DSGEs are perhaps most used for analyzing monetary policy. Denmark has since the beginning of the 80's had a fixed exchange rate regime, and as such the interest rate is set to maintain the peg to the Euro. Because of this, we commonly assume an exogenous interest rate in our models and do not include monetary policy reactions. DSGE models are an important source of inspiration in the development of MAKRO, but there are avenues where we look elsewhere for solutions. A key example is the modeling of uncertainty. DSGE models are by definition stochastic. In this project, we will generally not attempt to explicitly model decisions under uncertainty[1]. However, though the model is not stochastic, uncertainty plays an important role in household and firm decision making. Therefore household and firm behavior should ideally be modeled in such a way that their reactions to shocks are consistent with realistic behavior in an uncertain world. As stated, we wish to model both the short and the long run. Additionally, we want a model that is relevant to policy makers, i.e. a model with many institutional details, particularly on the fiscal side. It is our judgment that this can best be satisfied with a model without stochasticity. It is an important pragmatic decision to not model uncertainty directly. It is very difficult to handle stochastic problems mathematically correctly, without resorting to over simplifications to keep the model tractable. One example might be the Calvo pricing mechanism. Such simplifications are a major point of criticism of the dominance of DSGE models (see for example Romer, 2016 and Blanchard, 2017).

MAKRO is a non-linear model *without* stochasticity, instead of a log-linearized model *with* stochasticity. In this regard, MAKRO is more similar to a SEM. DSGE models are

[1] The exception is stochastic transitions to absorbing states, for example households' optimization with a probability of retirement or uncertainty of death.

typically log-linearized around a steady state, and model paths must therefore be interpreted as being close to a steady state. This is *not* the case with MAKRO. Instead we use a dynamic calibration process, which only requires that the system must move towards a stationary state in the long run[2]. This is an important detail when building a model with fully specified demographic changes or with a foreign demand that grows faster than the economy. These are major shocks to the economy, which are present in the baseline. The baseline can therefore not be interpreted as a steady state and the model must be calibrated outside steady state.

We believe the absence of uncertainty is an advantage for MAKRO, but there are of course disadvantages that come with it - namely that behavior related to uncertainty cannot be modeled directly. It is important to be aware of this limitation and to work around it where needed. For example, we include ad hoc risk premia in the arbitrage condition of the firms to get an equity premium that separates the returns on stocks and bonds, and we assume exogenous debt ratios of households and firms in the prototype model. Another example is the precautionary savings motive, which does not exist in models without uncertainty. Here, however, the same is true for DSGE models solved using log-linearization.

Within the 3 years of planned development, we will implement multiple approaches to expectations formation. We are currently experimenting with perfect foresight on one hand, and adaptive or static expectations on the other, but also more moderate choices e.g. based on sticky information (Mankiw and Reis, 2002) or bounded rationality (see for example Gabaix, 2016). Later on, it might also be possible to challenge the assumption of representative agents. In the current version of MAKRO there are two types of households (Ricardian and hand-to-mouth consumers) for each age group, and one representative firm for each production sector. As such, we already have a relatively large number of agents in the model, but further heterogeneity might for example be added by differentiating groups of households by life time income or education.

2 The prototype model - An overview

In the following we provide an overview of the prototype model that exists at the time of writing, while indicating in which direction we are developing each piece of the model. As such, the details are all preliminary.

2.1 Households

MAKRO is a cohort-based overlapping generations model with annual frequency and a life cycle which is 85 years long, starting at the age of 16. Household members share their income and assets at the start of each period. The size of the cohort aged a at the start of period t is given by $N_{a,t}$ and this quantity is exogenous and obtained from the data. All decisions are taken, and consumption occurs at the start of each period. Household members can die at the end of a period, so that next period the cohort aged $a+1$ contains the surviving members from the current cohort plus net migration of agents aged $a+1$. Apart from a fraction of

[2] In fact, we only require that agents' intertemporal budget constraints are satisfied.

hand-to-mouth consumers who have a rule of thumb savings decision, the only heterogeneity results from age. Our benchmark is the perfect foresight model.[3]

The household derives utility from consuming non durable goods, and from housing.[4] The core of the household problem is to choose an optimal consumption and savings path over the life cycle given a life cycle income path. With a frictionless access to finance the solution to this problem yields the usual consumption smoothing outcome which comes out of the intertemporal first order condition. In a set-up without bequests this first order condition looks as follows:

$$\frac{\partial U_{a,t}}{\partial C_{a,t}} \frac{1}{P_t^C} = \frac{1}{1+\theta} \frac{\partial U_{a+1,t+1}}{\partial C_{a+1,t+1}} \frac{1}{P_{t+1}^C} R_{a+1,t+1}$$

Consumption follows a flat trend over the life cycle thereby smoothing a hump-shaped income path. The same applies to the housing choice. The data, however, paints a different picture, with both consumption and housing tracking income, and the key element that allows our model to match the data (in addition to hand-to-mouth consumers) is the departure from frictionless finance, both in term of consumption finance and in terms of mortgage financing for houses.

In the model we introduce household banking for consumption smoothing purposes by forcing households to borrow from banks rather than having access to a risk free bond market. And in this banking relationship the cost of credit and the return on savings are modeled as a state dependent interest rate, $R_{a,t}(B_{a-1,t-1})$. This interest function is such that positive assets earn a return lower than the discount rate, and negative assets pay an interest greater than the discount rate. While we do not know the discount rate, this property is clear in the data on bank interest rates offered and charged to households. This feature then allows the consumption path to track income as closely as indicated by the data.

In addition to consumption finance, the model contains separate mortgage finance for houses. The mortgage rate is exogenous and lies in between the active and passive rates that affect consumption smoothing.[5] This property ensures that agents want to borrow to buy houses, and do so in excess of what they would if all rates were identical, as young agents will have negative savings and therefore pay a high borrowing cost. It is therefore a good decision for the household to allocate borrowing towards houses and not towards consumption. Old agents, on the other hand, have an incentive to downsize their house as the cost of mortgages is higher than the earned interest on positive assets.

One additional feature of the financial side of the household model is that mortgage finance does not extend to the full amount of the house but only to a maximum of 80% of its

[3] While individuals do not know whether or not they will be alive in the following period, due to the law of large numbers the cohort household knows with certainty how many will be alive then. Below we discuss alternatives to perfect foresight.

[4] There is also a labor force participation decision which is discussed in the labor market chapter. Non housing consumption is the result of a CES nest which relates to the input-output structure of the data detailed below. We also have habit formation in the model.

[5] Mortgage banks are separate from retail banks and currently operate only on housing in the model. In the future they will extend operations to corporate structures.

value. Therefore young agents have to borrow from the bank at a high cost to buy their initial housing. This in turn ensures that housing is relatively low in early ages and grows over the early part of the life cycle.

The last elements of the model are the death probabilities and the bequest motive. This is introduced in a way that is consistent with the cohort based approach. In the cohort model, death probabilities are implicit in the size of the population over the life cycle. Death probabilities and bequests, however, are individual effects which affect optimal intertemporal choices in a non-cohort manner. Here, when making an intertemporal decision, the household takes into consideration the size of next period´s cohort for the surviving fraction of the current cohort, and only the size of the current cohort for the warm glow bequest motive accruing to the dying fraction of the current cohort.

The bequest model highlights the timing underlying optimal decisions. At the start of a period (January 1st) agents are alive or dead and this is determined before any decisions for that period are taken. Agents who are alive receive their income for the period just starting and also draw from their pooled assets and interest earned carried over from the previous period to make their consumption and investment decisions. Agents who are dead have their carried-over assets and interest liquidated, taxed, and distributed among their heirs, and these quantities are pooled only from the dying part of the previous period cohort, even though they are distributed in a predetermined manner among all currently living agents.

2.2 Firms

There are s sectors where output is produced with different technologies. In each sector firms maximize their value under monopolistic competition, which is equivalent to minimizing costs and setting prices as a markup over marginal production costs. Prices are assumed to be sticky, with Calvo price setting used as benchmark in the test version of the model[6]. Different types of sticky information will be tested in later versions. The chapter on expectations will give more detail on this.

Currently the model has five sectors: housing, construction, manufacturing, private services, and public services, and within each sector firms have four different inputs: materials, labor, machine capital, and building capital. The inputs of each type in each sector are chosen as to minimize costs given a nested CES-technology with constant returns to scale[7]. The problem of the firm includes quadratic installation costs, and differences between the actual and the tax deductible depreciation rate. There is also corporate debt financing as part of the capital structure of the firm.[8] The value of the firm is the discounted value of its future dividends.

[6] Calvo price stickiness means that only a fraction of the firms in each sector can reset prices in a given year. Remaining firms keep their old prices. Forward looking firms take this into account when setting prices.

[7] Later versions of the model will be expanded with an agricultural and oil extracting sector with a non-renewable input.

[8] The firms' debt value equals an exogenous constant share of the value of the capital stock. In the future we aim to have a micro founded capital structure choice.

2.3 Financial Intermediation

Our model of financial intermediation is built for a number of specific purposes. On the household side, the aim is a more realistic life-cycle savings and asset profile. In addition, the separation of mortgage financing from consumption finance allows for the interest rate differentials we observe in the data to affect the choices our model consumers make.

On the firm side, we view financing and the financial accelerator as a key element of cyclical fluctuations. In our set-up, firms have access to credit lines and the terms of these credit lines depend on the collateral (the capital stock) the firm has. This leads to an accelerator mechanism whereby borrowing to invest in capital increases collateral which improves the terms of credit leading to further borrowing. An important feature of the model is that, in order to have a non explosive accelerator mechanism, the collateral effect must be subject to sufficient diminishing returns.

Our set-up is currently aimed only at the economics of financial intermediation. For that purpose our banks exist only as zero-profit trading and clearing houses with no production inputs. Their profits are zero due to an exogenous fixed cost, and this sets their economic size in National Accounting to zero. We may also analyze credit policy in this context.

2.3.1 Households

As mentioned in the household summary, retail banks engage in financial contracts with households for consumption smoothing purposes. The banking relationship is contained in

$$r_{a,t} = r_{t+1} - w_{a,t}^{banks} \lambda B_{a,t}$$

where $w_{a,t}^{banks}$ denotes the share of assets that is allocated to the bank rather than to the purchase of other savings objects, and $\lambda > 0$ is a bank financing cost parameter. Here positive liquid assets (deposits) earn a rate of return lower than the interbank rate, and negative assets (loans) pay a rate higher than the interbank rate. In the first order condition for savings, the marginal rate of return is written

$$R_{a+1,t+1} = 1 + r_{a,t} + B_{a,t} \frac{\partial r_{a,t}}{\partial B_{a,t}}$$

Banks trade at the rate r and borrow from and lend to households. They profit from both types of transactions, as they earn $r - r_{a,t}$ on their deposits, and $r_{a,t} - r$ on their loans. Their profits are then given by

$$\pi_t^{Bank} = \lambda \sum_a \left[w_{a-1,t-1}^{banks} \times B_{a-1,t-1}^2 \times N_{a-1,t-1} \right] - \kappa_t^{bank}$$

and here in our prototype we allow for ad hoc bank operational costs κ_t^{bank} to vary with time in order to keep profits at zero with a constant household banking cost λ. We either set both λ and κ to zero, to shut down the banking channel, or to positive values to match the active/passive interest rate dispersion observed in the data. For example, the June 2015 interest rate on bank deposits by non financial corporations was 0.227% and for deposits by households was 0.443%. On the other hand, the interested rate charged on bank loans to households was 3.569% for housing and 5.605% for non housing. The loan rate on housing mortgage bonds was, however, only 1.084%. Finally, credit policy analysis may be done by changing λ.

Regarding the mortgage bank, what we have is a linear contract and a linear technology so banks profits and policy parameters are not specified. The mortgage sector is reduced to an exogenous interest rate.

2.3.2 Firms

The modeling of households applies also to firms. Firms can use a credit line with the bank to obtain a cash injection $X_t > 0$ or save into a deposit $X_t < 0$, the accumulation of which can result in the existence of a permanent bank debt which they manage, as long as there is advantage in having such a debt. This can come from the bank being able to interact with the firm and provide cheaper financing than the market alternative due to a proximity relationship. Under this assumption the firm solves the following problem: choose capital stock and bank debt levels to maximize

$$V_t = max_{\{D_t,K_t\}} \left\{ p_t K_{t-1}^{\alpha} - P_t^I I_t + X_t - R_{t-1}^F D_{t-1} + \beta_{t+1} V_{t+1} \right\}$$

where $D_t = D_{t-1} + X_t$ and $R_t^F = R^F(D_t, K_t)$.

Because of the linearity of debt, the requirement must be that there are strong diminishing returns to collateral, or else the incentive to borrow to accumulate capital which in turn makes borrowing cheaper leading to a further increase in capital accumulation, could make the model unstable and unsolvable. Consider an example with $R^F = r + \lambda D - \phi K^{\theta}$. This problem has a well behaved solution if $\alpha < 1$ and $\theta < 1/2$. Not only that, the collateral effect must also be small in absolute size (a small value of ϕ) in order to ensure non negative bank profits. In this model it is easy to show that the bank stands to make a profit in the case without financial accelerator, that is, when $\phi = 0$. By continuity, the model solves also for small values of this parameter. But most importantly, as long as there are enough diminishing returns to collateral there will be a well behaved unique steady state.

On a final note, this type of modeling is pervasive in the literature and appears for example in DSGE models developed by economists at National Bank of Denmark.

2.4 The public sector

A key purpose of the model is to forecast the structural budget balance and to evaluate the effect on it of changes in policy and other variables. The Ministry of Finance calculates the structural budget balance by projecting the actual budget balance and making adjustments based on the estimated gaps in output and employment, as well as correcting for particular short term budget effects such as fluctuations in tax revenues from oil and gas production. This approach requires a good forecast of the budget balance as well as a good estimate and forecast of the output gap. The estimate and forecast of the output gap is discussed in its own chapter. Here the focus is on the government's budget balance.

The government's budget balance is government revenue net of government expenditure, including servicing of any government debt, and we model both revenue and expenditure in detail. For many policy reforms the Ministry of Finance uses specialized models based on micro data to calculate changes in individuals' marginal tax rates as well as in average tax rates. These micro estimates can be fed into our model to calculate the budget effect including general equilibrium effects.

The two largest components of government revenue are direct and indirect taxes, and we model both explicitly. Remaining revenue is a residual budget item. Direct taxes consist of personal income tax and corporate tax. Income tax is currently modeled as a flat rate on all income. This will be refined in later model versions. Income consists of wages and government transfers. Corporate tax is a flat rate on the firms' surplus. Indirect taxes consist of production taxes, VAT, duties, and customs. The Input/Output data gives imputed indirect tax rates for all the different Input/Output cells. Customs revenues are transferred to the EU and modeled that way.

Government expenditure consists of government consumption, transfers, and a residual[9]. Government consumption is endogenously given by a policy rule modeled as a gradual approach to a constant debt to GDP ratio. Transfers are divided into transfers to retirees, to people outside the labor force, and to the unemployed, plus a residual post. Transfers to a specific group are the sum of the age specific transfer rate times the size of the specific group of this age. The determination of the number of people in each age group that are retired, unemployed, or outside the labor force, is explained in the chapter on the labor market.

2.5 Foreign demand

Denmark is a small open economy and as such, developments in the international economy have a major domestic impact. Therefore, how we model foreign trade is extremely important. The demand for imports stems from the optimization problem of domestic agents and is firmly grounded in our micro structure. Foreign demand for Danish exports, on the other hand is currently modeled using the Armington model, and this is a framework which we aim to improve on as it has several caveats.[10]

The first important caveat concerns the price elasticity of the export demand function. In the Armington model, the size of Danish exports relative to the foreign demand level is determined by the Danish export price relative to the foreign price level with a given constant elasticity.[11].

[9] A part of the expenditure for government consumption is the depreciation of capital. As investments and not depreciation is part of the budget balance - public investments net og depreication is captured as part of the residual.

[10] Other research and policy papers using the Armington model are Burriel, Fernández-Villaverde, and Rubio-Ramírez, 2010 and Pedersen and Ravn, 2013.

[11] For example, if the elasticity is -2 an increase in the Danish export price of 1 per cent would then lead to a fall in Danish exports of 2 per cent.

There has been some debate about the size of this elasticity in Denmark but single equation estimations typically give an export elasticity that is relatively small (2.5 or smaller). (Nagel, 2015; Finansministeriet, 2012). At the same time, it can be argued that Denmark cannot deviate from the long-term foreign price level for too long without significantly damaging exports. If the domestic price level were to suddenly increase relative to the foreign price level (given the fixed exchange rate regime) the short term impact on exports would likely be small, but in the long term goods in the most competitive markets would likely loose market shares if the price changes were permanent. Such considerations suggest the long-term export elasticity is significantly higher than in the short term, implying the simple Armington model is not rich enough to explain data.

Another caveat of the classic Armington specification is the absence of a *scale effect*. The fact that large countries trade large quantities in the world market at the same price as small countries which trade small quantities, is a problem for the standard Armington model of export demand, which relates quantity exported to the ratio of domestic to foreign prices with a given elasticity. To understand this note that in a textbook open economy without exports but with an exogenous interest rate, the capital/labor ratio is constant and only real wages are defined as the domestic price level is indeterminate. Here, doubling the labor force will double the capital stock and double output. Once we add a foreign demand for exports using a standard Armington specification, the nominal price of domestic goods is determined and doubling the labor force does not result in a doubling of domestic output as a doubling of output produced does not result in a doubling of exports demanded. We therefore need to add scale to the Armington export demand, something which is also the subject of recent research from the ADAM group (Temere and Kristensen, 2016a,b).

2.6 Labor market

Our model of the labor market must be able to reproduce the level and behavior of wages, age specific unemployment rates, and age specific labor force participation. Of particular importance is the ability of the model to reproduce the fact that adjustment over the cycle falls mainly on employment rather than on wages. In order to achieve this our models include standard features of the labor market such as wage rigidity and labor adjustment costs.

Currently, household members can be in one of four labor market states: employed, unemployed, out of the labor force, or retired. In most of the following discussion we treat the number of agents out of the labor force as exogenous. The number of retired agents by age and over time is currently always exogenous and calculated outside the model.

We have several models of the labor market. All of them contain a labor demand curve which is the marginal product of labor schedule from the optimization of the firm. The supply side is more involved. It is straightforward to introduce separable disutility of labor in the utility function and generate an upward sloping supply curve. That generates an equilibrium but not meaningful unemployment. The simplest model of unemployment we use has a Phillips curve. The Phillips curve can be viewed as a transformation of the disutility of labor schedule in the supply and demand model. It differs from that model in that it contains an unemployment target which anchors the equilibrium obtained.

We are also developing a search and matching model of the labor market. The basic mechanics of the search and matching models differ from standard supply and demand models. Once workers join the labor force they are either employed or unemployed as a result of both search and matching frictions which are, in the simpler versions of the model, outside their control. Market clearing then occurs via free entry of vacancies and via a bargaining process that determines the wage which partitions the total surplus from a job.

2.7 Production and the Input/Output-system

The Input/Output system connects the demand and supply side of the model. Production in each domestic sector is the sum of deliveries from this domestic sector to all demand sources.[12] In the current version there are five sectors: housing, construction, manufacturing, private services, and public services.

The demand components consists of material and investment demand from the firms, consumption demand from the households and government and exports. Aggregate private consumption is the aggregate consumption of all cohorts in a given year. The total private consumption is divided into consumption groups according to the structure of a nested CES tree to give the demand for different consumption groups that maximizes utility. Government consumption consists in the current version of the model of only one government consumption group. As explained in the firm chapter each sector has an aggregate demand for material components and investments. Investments consist of three different types: machinery, buildings and inventory. Demand for material and investments are sector specific and has a sector specific input composition. In the current model version there are 24 demand components.

Each demand component has deliveries from each sector. Deliveries from each sector consists of a mix of domestic production and imports. The demand components are CES aggregates of the deliveries from the sectors. Typically the elasticities of substitution are 0 in these nests implying fixed input shares.

For each delivery, from a sector to a demand component, the domestic sector is in competition with a corresponding foreign sector. The choice between domestic production and imports are for each delivery handled with an Armington model. This imply that the delivery from a sector to a demand component is a CES aggregate of domestic production from this sector and imports from a corresponding foreign sector. The deliveries from the 5 domestic production sectors and corresponding 5 import groups, i.e. 10 supply components to the 24 demand components, are the core of the model's Input/Output matrix.

2.8 Data, calibration and baseline projection

The main data input is ADAM's data bank which consists of detailed national accounts supplemented with other data from Statistics Denmark. The data bank is extended with a number of short term forecasts from the ministries of finance and economic affairs. Our model uses

[12] Behind this definition lies the assumption that all deliveries from the same sector are the same product and hence have the same price net of taxation.

the most recent forecast year as its last calibration year, ensuring this way that it is in accordance with both the national accounts and the government's short term forecast.

The data bank extended with the short term forecast does not contain detailed cohort-specific data that can match our overlapping generations model of consumption. Nevertheless we make use of the ministries' population projection, which forecasts the number of people in each socioeconomic group for each cohort. Statistics Denmark provides an impressive collection of microdata, much of it based on tax registries, such as individuals' taxable wealth, debt, and income. To calibrate the model to the last year of the short-term forecast, historical cohort data is used together with the forecasted aggregates to impute cohorts' wealth, debt, income, and consumption. The Danish data registries are a very rich source of information and will be the foundation for future model expansions regarding the financial side of the household.

The model is dynamically calibrated outside steady state to match the output and labor market gaps estimated by the Ministry of Finance. All variables are calibrated to be in exact accordance to the data. This gives overfitting in the way that calibrated parameters change every year with no theoretic or empirical explanation. In order not to have the overfitting contaminate our forecast and impulse responses we do two things. First, we use dynamic calibration only for parameters which are of little importance to the marginal properties of the model. For example, in the CES-functions, the elasticities are estimates whereas the share parameters are calibrated. Second, we determine structural values for all dynamically calibrated parameters. How these structural values are determined is explained below.

The baseline projection starts where the short term forecast ends. In the baseline projection all calibrated parameters return to their structural levels at a pace empirically estimated. All other parameters are kept constant. The interest rate for EU money market and bonds, the growth rate of the export markets, the import and export prices, demographic changes, and all tax rates and other public policy rules are exogenous in the model. The ministries have projections for these, which are included in the baseline projection.

3 Structural levels and trends

Policy makers care about the structural levels of the economy, and how and why observed values of, among others, employment, productivity, and GDP deviate from such values.

The Ministry of Finance estimates structural values for the economy using a state-space and Kalman-filter approach, and we want to calibrate our model so that the initial gaps between structural and observed values in the base year correspond to the gaps calculated by the Ministry of Finance[13]. For that the model must fit both observed and structural data. An adequately calibrated "unconstrained" model yields actual levels.

We then calculate structural levels for a given year solely on the basis of that year's data,

[13]The Ministry of Finance's Kalman-filter approach calculates the historical developments in structural GDP, productivity, employment and labor force. See (in Danish) https://www.fm.dk/~/media/files/oekonomi-og-tal/fm-regnemetoder/opdatering-af-finansministeriets-beregning-af-konjunkturgab-og-strukturelle-niveauer.ashx?la=da

including actual initial stock values. We can do this removing all nominal rigidities from the model and setting productivity to its structural level. In addition, we can put foreign demand and prices as well as interest rates to their exogenous structural levels.

Defining structural levels has one advantage. It allows us to measure the model's speed of adjustment as it gives us a target towards which adjustment occurs. From a validation point of view, this is important, as we follow the literature and use empirical SVAR models and their impulse responses to measure such adjustment. Key parameters in our model will then be calibrated in order to minimize the difference between empirical and model generated impulse responses.

Defining 'structural' is a non trivial task. Structural levels can be seen as long-term levels or - more technically - as the levels that occur in the model when all or some of the modeled rigidities are turned off. We define them more broadly as *cyclically neutral levels*, which measure the state of the economy in the absence of cyclical fluctuations. We find that this definition appropriately encompasses the structural concept used by the ministries in their daily work.[14]

If structure is defined as the absence of a business cycle, we must then define what the cycle is, or better what induces it. Apart from the standard technology or preference shock device, typical small open economy business cycles are generated by exogenous fluctuations in foreign demand, foreign prices, interest rates, etc[15]. Nominal and real rigidities then help propagate these shocks throughout the economy.

Computing cyclically neutral levels in the macromodel is a complex task. As mentioned above, we are planning to calculate structural levels for a given year solely on the basis of that year's data, including actual initial stock values (capital, wealth ect.). We can do this by removing all nominal rigidities from the model and setting productivity to its structural level. In addition, we can put foreign demand and prices as well as interest rates to their exogenous structural levels.

An alternative way to compute cyclically neutral levels is by 'backcasting' the model. Suppose we were able to calibrate the model to provide a good description of the Danish economy for e.g. 2000-2017. With this calibration we would then use the model to calculate the structural development for 2000-2017, where productivity, foreign demand and prices, interest rates, etc. followed structural exogenous developments so that the economy did not receive economic shocks (for example by filtering all these exogenous time series). This would be a model-consistent dynamic estimate of the cyclical-neutral course of the economy. If 2017 is the start year of our model, this backcasted model's structural estimate of 2017 would be a good starting point for projections. Running the model beyond 2017 would yield an estimate of the future structural development. Note that stocks (capital, assets, etc.) at the beginning of 2017 would differ from actual stocks. The model stocks in 2017 would be the accumulation of structural flows from the entire period 2000-2017.

[14]The ministries must on a day-to-day basis position themselves in relation to where the economy is in the business cycle. And in projections, they must relate to the state's revenue and expenses during the economic movement out of the current cyclical fluctuation. The *structural balance* is the most central example of this.

[15]As mentioned before, Denmark has a fixed exchange rates regime. Active monetary policy (such as a Taylor-rule) is therefore *not* an issue when it comes to explaining Danish business cycles.

The main difference between the two methods is whether the stocks are actual or model-calculated "structural" values. The importance of this has to be investigated.

4 Expectations

As a starting point, our benchmark model is a deterministic perfect foresight one. Both these features are standard practice extreme simplifications of the world. They are used for convenience and to impose discipline on the modeling exercise as they allow us to focus on preferences, technology, market structure, and market frictions as the main determinants of observed behavior.

They are nevertheless strong assumptions, and following a number of literature refinements on the implementation of rational expectations and on full information, we are interested in studying selected departures from the perfect foresight environment. Of particular interest to us is the sticky information model of Mankiw and Reis (2002) which is an alternative to often used sticky price theories. The basic idea is that agents are rational given the information they possess, but that they only gradually receive the correct information about the state of the aggregate economy.

The first order condition of the household problem is useful to discuss what the deviation from full information means in our problem

$$\frac{\partial U_{a,t}}{\partial C_{a,t}}\frac{1}{P_t} = \frac{1}{1+\theta}\frac{\partial U_{a+1,t+1}}{\partial C_{a+1,t+1}}\frac{1}{P^e_{t,t+1}}R_{a+1,t+1}$$

Note that on the right hand side we have introduced an expected price rather than the actual perfect foresight price, which would be the case in the perfect foresight model. We then must add a rule for the formation of expectations:

$$P^e_{t,t+1} = F\left(P_t, other\right)$$

The way this equation is written becomes fundamental for the behavior of the model, in this case for the savings decision. There are additional technical details in the above equation. Future consumption must enter also as an expectation so that all future components of the budget constraint of the household are now subject to expectation formation processes, and this must be done in a consistent way. These are problems that have been studied and in the literature. The number of challenges, and of opportunities does not end here, as limited information is a problem that extends to all agents in the economy, possibly in different ways.

Departing from full information in our model is therefore a promising way to fill the gaps left unexplained by the standard full information benchmark.

5 Estimation and calibration

The empirical methodology must ultimately be guided by the model type at hand. At least two characteristics of the model and their implications for parameter estimation are worth noticing.

First there is no explicit uncertainty in the model. Instead, a number of variables exogenous to the model (foreign output growth, interest rates, etc.) are responsible for driving most of the endogenous response. Furthermore, following the tradition of CGE models, the remaining unexplained variation is captured in a number of parameters which are dynamically calibrated each year. This is different from DSGE models where parameters are held fixed (either from calibration or estimation) and where all fluctuations must be driven by specified structural shocks or so-called measurement errors. The lack of an explicit probability distribution over a set of stochastic processes implies that MAKRO has no likelihood function. Second, the model contains a very high number of equations due to the detailed multi-sector production structure and the overlapping generation age disaggregation. As a result, the dimensionality of the parameter space is much larger than in a standard DSGE model and a system estimation is therefore infeasible, even if the model was stochastic. This calls for an eclectic parameterization strategy that is dependent on the type of parameter considered. To focus the subsequent discussion, it is useful to classify model parameters into four categories:

1. First, a set of parameters are fixed in accordance to microeconometric studies or to ensure external consistency with established principles and assumptions of the model user. Examples include income and substitution elasticities of the labor supply.

2. Second, given the model structure, some parameters can - consistent with the model structure - be estimated in isolation and prior to running the remaining model. An example hereof is the nesting structure of consumer utility, which allows us to go directly after the elasticity of substitution in the resulting demand functions.

3. Third, a set of parameters are primarily responsible for the marginal behavior of agents but may not be covered by the first two categories, for example because they lack a clear microeconomic equivalent or consensus about the values. Still, these parameters will guide the endogenous response to exogenous changes and are thus crucial for the short-term dynamics. Examples of such parameters are those that control the intensity of frictions in the economy, for example such as price rigidities or adjustment costs of investment. We will calibrate these parameters so that the model impulses are reasonable, compared to those obtained from estimated VAR models.

4. Finally, a fairly large number of parameters are subject to dynamic calibration, for example share parameters to match input-output tables. Hence, these parameters are essentially treated as data historically. However, when making projections or policy analysis, it becomes important whether the parameters are assessed to be close to their structural level in the last year of the sample or not. This is in essence a filtering exercise and we intend to make use of the Kalman filter for this task. Importantly, these parameters' "residual-like" qualities also serve as an indicator for model misspecifications.

The distinction between parameter types is not always clear cut: For example, capital adjustment costs are important for investment impulse responses but can also be dynamically calibrated to match the observed capital stock. Also, the elasticities in the first and second category are clearly also important for the marginal behavior, and could be freed up set to match the VAR impulses. More generally, since most parameters will affect both steady state levels and marginal behavior, there are no definite answers on how to categorize them. However, the very high level of details in the model reduces the risk that model parameters are "reduced-form" parameters that contain unmodeled characteristics of the economy, thus providing the model users with more degrees of freedom. Finally, despite this eclectic approach we require the *overall* model dynamics to be reasonable for a given set of parameter values.

5.1 Empirical vs. model impulses

The workhorse for the calibration of the third set of parameters will be Vector Autoregressive models (VARs). They have the advantage of being an extremely flexible description of the covariance structure in the data, and therefore one of the preferred tools in empirical macroeconomics. Although the deterministic setup of the model in principle rules out a VAR representation, one can still think of a VAR model as being a reasonable approximation of the model equilibrium. In fact, many existing macroeconomic models use the impulses of VARs as a "sanity check" once the model is estimated/calibrated. We intend to go a step further and use the impulse responses of the VAR model to calibrate key parameters mainly responsible for the dynamic response of the model to the same shock. These parameters control the intensity of the frictions in the model economy and thus guide the marginal behavior in response to exogenous changes, something that a VAR model has a relatively high probability of identifying. Contrary to other parameters these frictions will not be subject to dynamic calibration but kept constant. They will however be subject to revision when significant changes are made to the model.

When studying empirical impulses, we prefer to identify shocks using multiple relatively sparse models, rather than a "one-size-fits-all" VAR. This has several advantages: First, considering the relatively short data series combined with the fact that the number of parameters increases quadratically in the number of variables in the VAR model, too large models will lead to an inefficient estimation of the parameters. Furthermore, larger models will require more restrictions to be imposed on the VAR model to recover the structural shocks from the residuals of the reduced-form VAR, restrictions which will be increasingly implausible as the dimension increases. On the other hand, multiple smaller VAR models can be "validated" by the empirical macroeconomic literature. Ultimately, the choice of specification rests on a trade-off between relatively efficient estimation on one hand and omitted-variable bias on the other hand. We perform a range of tests and robustness checks to ensure that the VAR models used are generally well-specified and that the recovered structural shocks are in fact that.

The VAR models will be estimated on quarterly data and the impulses subsequently annualized to match the frequency of the MAKRO model. The reason for this is that further restrictions need to be imposed on the estimated VAR model to identify structural (statistically orthogonal) shocks which can then be compared to the exogenous changes in the MAKRO model. We identify the shocks with a combination of contemporaneous restrictions (of which the Cholesky decomposition is a special case) and sign restrictions. Many of these restrictions are simply more reasonable at quarterly frequency. The VAR models will contain a core set of variables of particular interest plus some variables that contain information on the endogenous propagation of the studied shock. These variables will depend on which shock is conditioned on. For example, we would expect the effective exchange rate of the krone to play an important role in the monetary transmission mechanism but to be less relevant for studying fiscal policy.

5.2 Smoothing and dynamic calibration

As described previously, a significant number of parameters are calibrated dynamically in each period to match the observed data. An example could be the share of imported goods in the CES production in the different sectors, given the relative price indices and the elasticities. While the historical levels of these parameters are taken as given, the model user needs to decide on a reasonable future path when making projections. The calibrated value given the last year of data might not be the best projection, neither is the historical average necessarily. We think of the resulting time-series (of dynamically calibrated parameters) as reflecting both some underlying structural process and some noise, which ideally we would like to disregard. Such an exercise is a job for the Kalman filter. For the parameters in the CES nests of firm production and household utility specifically, the Kalman smoother can be employed to *simultaneously* smooth the time-series, indicate the adjustment from the actual to the structural level, and potentially yield an estimate of the CES elasticity of substitution.

We decompose these parameters in a structural component (which may contain a trend) and a short-term component (which may be persistent). As such, this will capture unmodeled phenomena such as the trend in import content in intermediate goods due to globalization or it may reflect business cycle dependencies. With respect to the latter, the persistency should ideally be "removed" by the model itself, thus mimicking the white noise assumption of measurement errors. This illustrates that the dynamic calibration to historical data is in itself an interesting exercise for validity reasons.

There are other areas where external smoothing of data can help in generating a path for parameters. For example, due to the detailed overlapping-generations structure, the model can match age-specific time-series such as bequests or employment and participation rates. However, since these series contain a certain amount of noise, it can be useful to smooth out the series before making projections (as is often done in the mortality-rate literature).

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