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MAKRO

Estimating the consumption system in MAKRO

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Summary

In this working paper, the elasticities of substitution between the various consumption components in MAKRO are estimated. Since the share parameters are unknown, a time varying process is specified. Thus, the Kalman filter can be used to simultaneously estimate the elasticities as well as the share parameters that express shifts in preferences for the individual consumption components. It is assumed that shifts in preferences can be decomposed into a structural and a cyclical part, with the former expressing long run trends that are allowed to deviate from a linear trend. The latter express short run fluctuations, e.g. because of cyclical changes in the economy. This setup results in plausible parameter
estimates and leads to a well-specified model.

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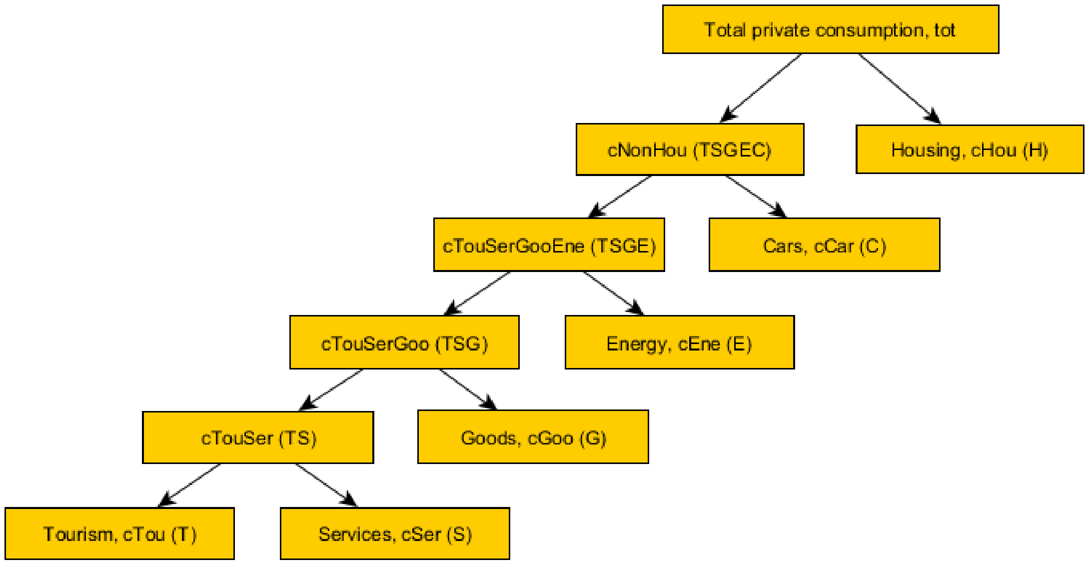
1 Introduction

In MAKRO, total consumption is divided into six different consumption components: tourism (cTou), services (cSer), goods (cGoo), energy (cEne), cars (cCar) and housing (cHou). The substitution between the different consumption components is determined by a price effect and a preference effect: The price effect expresses shifts in consumption due to shifts in the relative prices between two consumption components. The preference effect, on the other hand, expresses changes in consumption due to changes in preferences, e.g. increased demand for services that cannot be explained solely by changes in relative prices. The importance of the two effects depends on the elasticity of substitution between the different consumption components. Estimating this elasticity as correctly as possible is therefore important due to its implications for the model properties.

In this paper, we estimate the substitution between the different consumption components, with the exception of housing, which is estimated separately. We have chosen the time period 1983-2017 due to a potential data break and a higher quality of data. For the same reasons, this time period corresponds to what we use in the projection of structural parameters in MAKRO. We assume that consumption can be described by a nested CES consumption function, where there is substitution between two components in each nest. Since the share parameters that express preferences for the individual consumption component are unobserved, estimation requires making an assumption about their development over time. In this paper, we follow the method from [1] and use the Kalman filte to estimate the elasticity of substitution and the development of preferences simultaneously. The share parameter is decomposed into a structural component and a short run component. The former will capture long run trends in preferences while the latter will capture short run fluctuations, e.g. increased car purchases during an economic boom. We find that this setup results in plausible estimates of the elasticity of substitution as well as a well-specified model, based on a series of econometric tests.

The structure is as follows: Section 2 first presents the structure and the estimated model. We also discuss the methodological considerations as well as alternative specifications that have been tried previously. The estimated elasticities and share parameters are shown in Section 3. Section 4 summarizes.

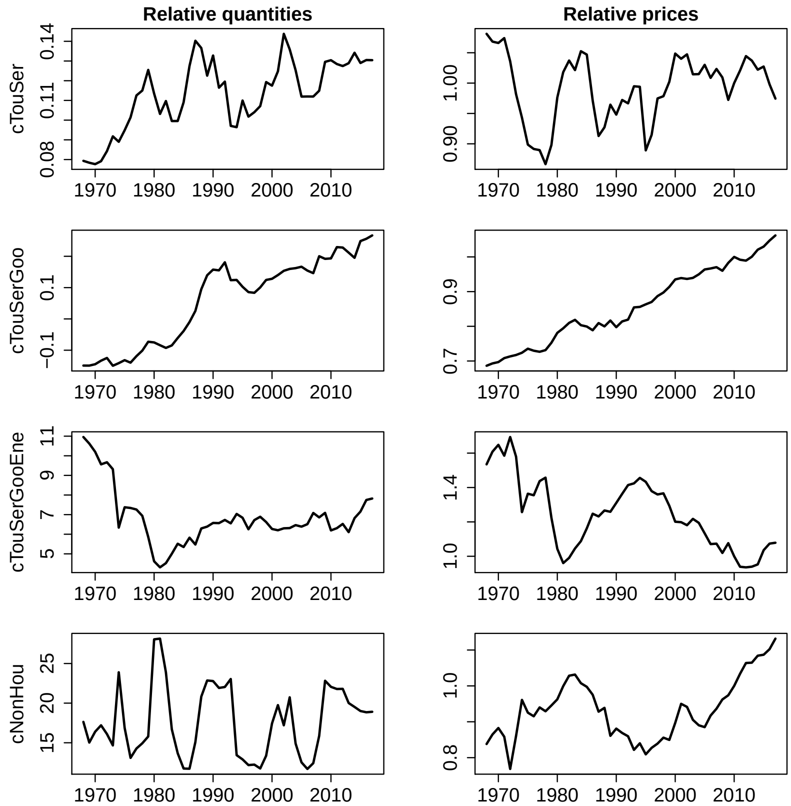


Figur 1: Nest structure

2 Nest structure and model specification

2.1 Nest structure

Total private consumption in MAKRO is divided into the six different consumption components mentioned in the introduction and can be written as $C_t P_t^C = \sum_a C_{at} P_{at}^C$, where $a = (cTou, cSer, cGoo, cEne, cCar, cHou)$. In the present paper, we estimate the substitution between all consumer goods except housing, which is estimated separately. We use a nested CES consumption function, substituting between two consumption components at a time, as this allows for different elasticities of substitution in each nest. The selected nest structure is illustrated in Figure 1. The individual nests are $u = (cTouSerGoo, cTouSerGoo, cTouSerGooEne, cNonHou)$, where e.g. cTurTje indicates the nest consisting of tourism and services. In the nests, the price is formed as a Paasche price index and the quantity demanded is formed consistently with an assumption that the nested value is equal to the sum of the value of the two consumption goods in the nest. The data series are from MAKRO's database and the time period 1983-2017 has been chosen, consistent with the period used in the projections of structural parameters in MAKRO. This time period is also preferred due to a structural break in energy consumption in the early 80s following the two oil crises. The data series used are shown in Figure 2. As a robustness check, we have also estimated the elasticities with using data from 1968. These results are shown in the Appendix.



Figur 2: Relative quantities and prices of the nest structure used. Both are shown as the first input in the nest relative to the second input, e.g. tourism relative to services in the first row.

2.2 Model specification

The utility function is given by:

$$C_{cu,t} = \left[\sum_j (\mu_{j,t} C_{j,t})^{\frac{\sigma_{cu}-1}{\sigma_{cu}}} \right]^{\frac{\sigma_{cu}}{\sigma_{cu}-1}}. \tag{1}$$

j indicates the two consumption components in the individual nests, e.g. cTur and cTje in the lower nest. t indicates the time period. We call $\mu_{j,t}$ a share parameter and expresses the preference for $C_{j,t}$, which can be time-varying. The elasticity of substitution is constant and given by σ_{cu} in nest u. If it is assumed that consumers are cost-minimizing, the following relative (log)demand in each individual nest can be derived on the basis of (1)[1]:

$$s_{cu,t} = (\sigma_{cu} - 1)\,\mu_{cu,t} + (1 - \sigma_{cu})\,p_{cu,t}, \tag{2}$$

[1] We estimate the relative demand rather than two equations simultaneously, as we have previously experienced problems with near-perfect multicolarity between the price of one consumption good and the generated price index.

where $s_{cu,t}$ is the relative budget share in logarithms in nest u, [2] $\mu_{cu,t}$ is the relative share parameter in logarithms in nest u and $p_{cu,t}$ is the relative price in logarithms in nest u.[3] The implications of time shifts in the share parameter depend on whether σ_{cu} is above or below one: When $\sigma_{cu} < 1$, an increase in $\mu_{cu,t}$ (relative increase in the preference for the first consumption component in the nest) causes a decrease in $s_{cu,t}$ (relative decrease in the consumption of first input in the nest). When $\sigma_{cu} > 1$, an increase in $\mu_{cu,t}$ causes an increase in $s_{cu,t}$. This emphasizes the importance of estimating $\mu_{cu,t}$ and σ_{cu} simultaneously, which requires assuming a process for $\mu_{cu,t}$, as it is unobserved for the econometrician. Based on the data used, this process must allow for structural shifts, as it e.g. is seen in the substitution between tourism services and goods, where the relative budget share has increased with an approximately linear trend towards less relative consumption of goods. Therefore, the chosen specification must allow for trends. In the nest between tourism-service-goods-energy and cars, the relative budget share has been constant in the long run, but contains significant short or »medium run« fluctuations, potentially reflecting the business cycle. Therefore, the chosen specification should allow for short run fluctuations that can be explained neither by changes in relative prices nor structural shifts in preferences. Based on these considerations, we decompose $\mu_{cu,t} = \mu_{cu,t}^{Structural} + \mu_{cu,t}^{Cyclical}$, where $\mu_{cu,t}^{Structural}$ describes structural fluctuations and $\mu_{cu,t}^{Cyclical}$ describes cyclical fluctuations and other temporary fluctuations which are not due to changes in prices. The processes are given by:

$$\Delta\mu_{cu,t}^{Structural} = \Delta\mu_{cu,t-1}^{Structural} + \eta_{cu,t}, \qquad \mu_{cu,t}^{Cyclical} = \sum_{i=1}^{p} \varphi_{cu,i}\mu_{cu,t-i}^{Cyclical} + \varepsilon_{cu,t}, \tag{3}$$

where $\eta_{cu,t} \sim N\left(0, \Sigma_{cu}^{\eta}\right)$ and $\varepsilon_{cu,t} \sim N\left(0, \Sigma_{cu}^{\varepsilon}\right)$. The number of lags in $\mu_{cu,t}^{Cyclical}$ is selected based on a criterion of no autocorrelation in $\varepsilon_{cu,t}$. The parameter $\varphi_{cu,i}$ is restricted to the interval ± 0.9 to obtain a clearly stationary process with a mean value of 0.[4] The resulting process for $\mu_{cu,t}$, will resemble a trend/cycle-decomposition known from the HP filter. At the selected specification, $\mu_{cu,t}^{Structural}$ will express structural preferential shifts between the two consumption components that is allowed to deviate from the often applied assumption of a linear trend. $\mu_{cu,t}^{Cyclical}$ reflects cyclical or short run changes

[2] The use of budget shares rather than relative quantities has the advantage that it results in smaller estimation errors if there are correlated measurement errors on both the right and left side.

[3] In the first nest they will e.g. be $s_{cTouSer} = log\left(\frac{P_{cTou}^{C}}{P_{cSer}^{C}}\frac{C_{cTou}^{C}}{C_{cSer}^{C}}\right)$, $\mu_{cTouSer} = log\left(\frac{\mu_{cTou}}{\mu_{cSer}}\right)$ and $p_{cTouSer} = log\left(\frac{P_{cTou}^{C}}{P_{cSer}^{C}}\right)$.

[4] By allowing $\varphi_{cu,i}$ to vary freely, $\mu_{cu,t}^{Cyclical}$ is most often estimated as a Random Walk. This is a well-known problem: a Random Walk is a good description of many time series and means that less emphasis is placed on prices. We do not find this plausible from an economic point of view.

in preferences, potentially due to the business cycle. How sluggish $\mu_{cu,t}^{Structural}$ moves depends on the relative (inverse) signal-noise ratio, $\lambda \equiv \frac{\Sigma_{cu}^{\varepsilon}}{\Sigma_{cu}^{\eta}}$. $\lambda \to \infty$ corresponds to a linear trend assumption and $\lambda \to 0$ corresponds to the case, where all changes in relative budget shares are due to structural shifts in preferences. In the estimation, we use the procedure described in [1], where the Kalman filter is used is the following way: First, λ is freely estimated by maximum likelihood and then it is subsequently calibrated for a grid of values in the range $\lambda \in [50; 1000]$. The value that maximizes likelihood and results in a well-specified model is selected. A well-specified model is required to have no autocorrelation in the residuals as well as meeting the conditions of the NIS test.[5]

We tried several different processes for $\mu_{cu,t}^{Structural}$, including a Random Walk-assumption, which however turned out to result in non well-specified models in many estimations (including residual autocorrelation problems). We have also estimated models in error correction form in the same way as [1], which generally results in well-specified models. However, the parameter estimates were estimated with a much higher uncertainty and the estimates were often of implausible magnitude. Furthermore, the likelihood value was also significantly lower than the equivalent for the »static« model used here.

3 Results

This section presents the estimated CES elasticities and share parameters using the state space model consisting of (2)-(3). The results are shown for the time period 1983-2017 and the results for the time period 1968-2017 are shown in Appendix A. The elasticity of substitution in the first nest between services and tourism is estimated to be 1.25 (Table 1). Services and tourism are thus substitutes, as the elasticity is greater than one. Conversely, tourism-service and goods are compliments (although close to Cobb-Douglas), as the elasticity is marginally less than 1 (0.94, Table 1). The elasticity to energy consumption is relatively low at 0.26. At other specifications, we have estimated it to be low and even 0 when the entire time period is used, which is why we consider the estimate to be qualitatively robust. Finally, the elasticity to cars is estimated at 1.04 and thus very close to Cobb-Douglas. However, it is estimated with significant uncertainty and is by other specifications estimated to be lower (around 0.2-0.5, results not shown here). Despite the fact that the standard errors on the parameters are relatively high, all elasticities are significantly greater than 0. One should note that this high uncertainty regarding the estimates is primarily due to the uncertainty behind the trend specification. If the (unknown) estimated process for the preferences is instead assumed to be correct, the standard deviations will be much lower. Only for energy that

[5]The NIS test is a test of whether the residuals are well calibrated and is often used in the Kalman filter literature.

	cTouSer	cTouSerGoo	cTouSerGooEne	cNonHou
σ	1.25 (0.34)	0.94 (0.33)	0.26 (0.08)	1.04 (0.48)
φ_1	0.86 (0.19)	0.89 (0.13)	NA (NA)	0.89 (0.19)
φ_2	NA (NA)	-0.13 (0.07)	NA (NA)	-0.21 (0.15)
nlags	1	2	0	2
Observations	35	35	35	35
Likelihood	64.92	102.81	81.47	37.83
λ	950	1000	100	800
Autocorrelation	[0.47]	[0.22]	[0.39]	[0.60]
Heteroskedasticity	[0.66]	[0.03]	[0.25]	[0.08]
Normality	[0.27]	[0.40]	[0.86]	[0.09]
NIS	0.89	0.86	0.84	0.87

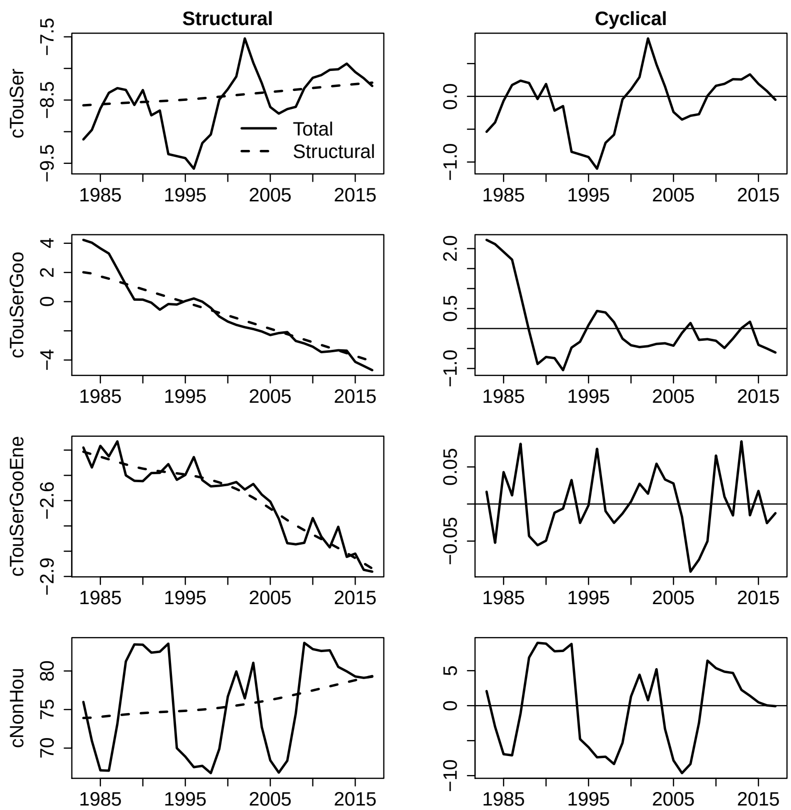
Tabel 1: Estimated results. Terms in parentheses are bootstrapped standard errors and brackets are p-values. nlags indicates the number of autoregressive lags, $\varphi_{cu,i}$. The critical value for the NIS test is [0.68; 1.37] at a 10% significance level.

the elasticity is significantly different from one, which may be an argument for applying a Cobb-Douglas assumption in the other nests if it has other model advantages.

For all four nests, we obtain a model without autocorrelation in the residuals at a 10% significance level and the NIS test for filter misspecification lies within the confidence interval at a 10% significance level. This is also confirmed by a residual and autocorrelation plot in Appendix B, where there do not appear to be significant outliers or autocorrelation in the residuals. However, it may indicate that there are heteroskedastic residuals in some of the nests (see the test for heteroskedasticity in Table 1). Heteroskedasticity invalidates the fixed-design bootstrapping procedure used and the bootstrapped standard errors may therefore contain measurement errors, which a block-bootstrapping procedure could potentially remedy.[6] For now, however, our primary interest lies in the point estimates, but another bootstrapping procedure could be interesting to try out as a robustness check.

Due to misspecification, free estimation of the signal-noise ratio was not been preferred in any nest (i.e. all preferred values of λ in Table 1 are chosen from the calibrated values in the grid). This shows that a calibration of the smoothing parameter to a certain value can often be preferable, as it provides a more well-specified model and higher likelihood. The preferred value for λ is relatively high in all nests except the tourism-service-goods versus energy nest. This means that the underlying process for $\mu_{cu,t}^{Structural}$ is fairly close to a linear trend with some gradually bending off. This is also

[6] We have not tested this bootstrapping procedure in our estimates yet. The idea would be to resample the residuals in blocks rather than individually. In this way, some of the temporal structure is retained in the data.



Figur 3: Decomposition of the share parameter, $\mu_{cu,t}$, into a structural, $\mu_{cu,t}^{Structural}$, and a cyclical, $\mu_{cu,t}^{Cyclical}$, component.

seen by Figure 3. In all cases, the filtered cyclical components are mean reverting, although with different persistence. Short and medium run fluctuations (up to about 5 years) - captured by the cyclical component- are described by an AR(1) process in the first nest and an AR(2) process in the second and last nest. In the nest containing energy consumption, no lags have been needed to ensure that no autocorrelation in the residuals is present. This may well be reflected in the lower value of λ, which allows the structural part to capture more of the fluctuations in the medium run, thus decreasing the need for persistence in the cyclical component of the preference effect.

4 Summary

We estimate the consumption system in MAKRO using the Kalman filter, analogous to the estimation of the elasticities of substitution in the production functions in MAKRO. Data from MAKRO's database has been used covering the period 1983-2017. A static regression is used, where the share parameter is decomposed into a structural I(2) process and a cyclical AR(p) process. The estimated elasticities are of plausible magnitude but determined with some uncertainty, which is not different from other estimates with uncertainty about the trend specification. All preferred estimated models are well specified and live up to a range of econometric tests.

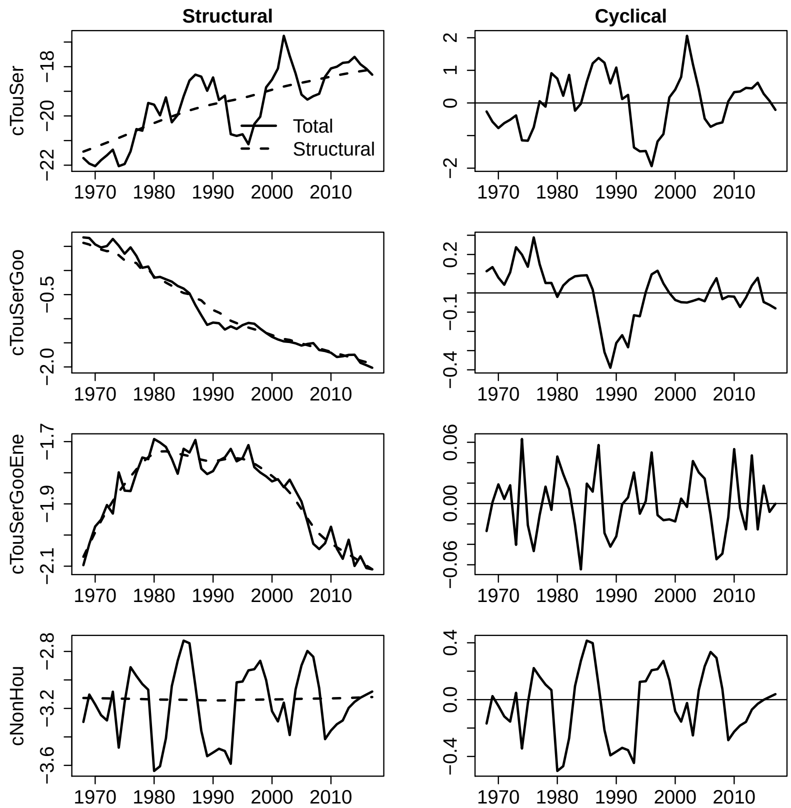
Litteratur

[1] A. F. Kronborg, C. S. Kastrup, and P. P. Stephensen. Estimating the Constant Elasticity of Substitution when Technical Change is Time-Varying: A Kalman Filtering Approach. Working paper, DREAM, November 2019.

A Another time period

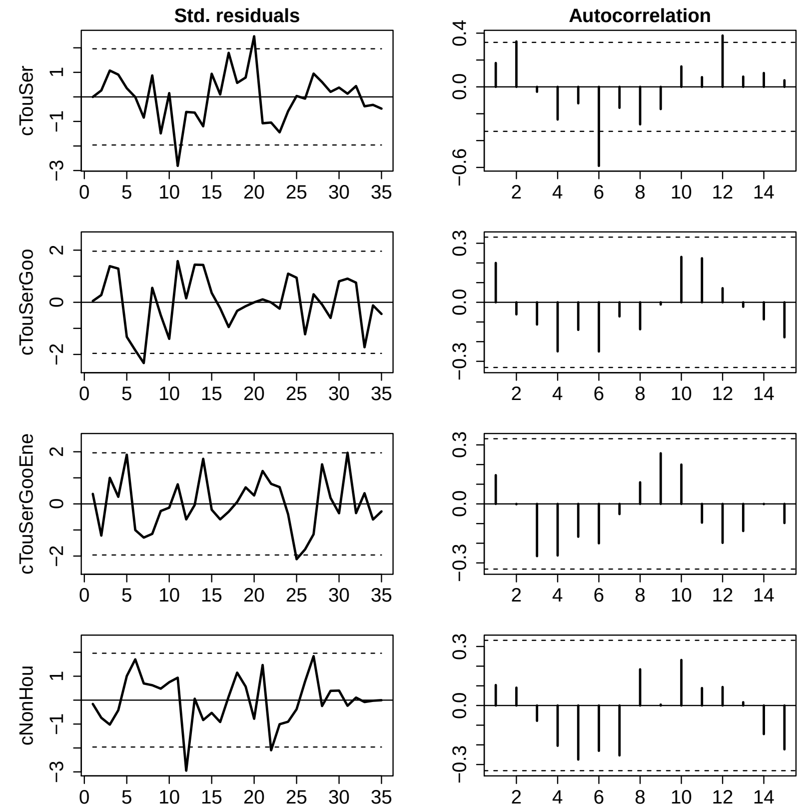
	cTouSer	cTouSerGoo	cTouSerGooEne	cNonHou
σ	1.11 (0.29)	0.71 (0.26)	0.00 (0.06)	0.06 (0.40)
φ_1	0.89 (0.27)	0.89 (0.08)	NA (NA)	0.88 (0.19)
φ_2	NA (NA)	0.09 (0.08)	NA (NA)	NA (NA)
φ_2	NA (NA)	0.01 (0.07)	NA (NA)	NA (NA)
nlags	1	3	0	1
Observations	50	50	50	50
Likelihood	98.36	148.61	120.24	50.40
λ	262	300	22	1015
Autocorrelation	[0.87]	[0.15]	[0.80]	[0.44]
Heteroskedasticity	[0.95]	[0.61]	[0.77]	[0.83]
Normality	[0.62]	[0.98]	[0.67]	[0.11]
NIS	0.90	0.85	0.81	0.95

Tabel 2: Estimated results for the period 1968-2017. Terms in parentheses are bootstrapped standard errors and brackets are p-values. nlags indicates the number of autoregressive lags, $\varphi_{cu,i}$. The critical value for the NIS test is [0.68; 1.37] at a 10% significance level.



Figur 4: Decomposition of the share parameter, $\mu_{cu,t}$, into a structural, $\mu_{cu,t}^{Structural}$, and a cyclical, $\mu_{cu,t}^{Cyclical}$, component. The time period used is 1968-2017.

B Residual plots



Figur 5: Standardized residuals and autocorrelation plot.